OLIMPIADA DE MATEMATICĂ

FAZA LOCALĂ – 18.02.2017

Clasa a VII-a- Bareme si solutii

1.

dacă p impar, iar r şi q pare, obţinem 987 pătrat perfect (fals), deci toate sunt impare .……1p

pentru orice *a* natural, avem $a^2 \in \{M_3, M_3+1\}$, $a^2 \in \{M_5, M_5+1, M_5+4\}$ …………..…....1p

verifică doar $p^2 \in M_3$, $q^2, r^2 \in M_3+1$ ……………………………………..…………………1p

obţinem $p = 3$, $q^2 + r^2 = 986$ ……………………………………………………..…………1p

verifică doar $q^2 \in M_5$ …………… …………..……………..……………………..…….……1p

obţinem $q = 5$ şi $r = 31$……….…………………..…………………………………….....…...1p

deci *p*, *q*, *r* pot fi orice permutare a tripletului (3, 5, 31) ………………………….……….1p

2. Ecuația se poate scrie : $(x-2017)(y-2017)=2017^2$……………………..…………….1p

2017 este număr prim $\Rightarrow D_{2017^2}=\{-2017^2, -2017, -1, 1, 2017, 2017^2\}$…………………………1p

$x-2017=-2017^2, y-2017=-1 \Rightarrow x=-2016\cdot 2017, y=2016$………………………..1p

$x-2017=1, y-2017=2017^2 \Rightarrow x=2018, y=2017\cdot 2018$……………….……...…..1p

$x-2017=2017, y-2017=2017 \Rightarrow x=2\cdot 2017, y=2\cdot 2017$……………………….……1p

$x-2017=-2017, y-2017=-2017 \Rightarrow x=0, y=0$, nu convine………………………...1p

Multimea soluțiilor este:

$(x,y) \in \{(2\cdot 2017, 2\cdot 2017), (2018, 2017\cdot 2018), (2017\cdot 2018, 2018), (2016, -2016\cdot 2017), (-2016\cdot 2017, 2016)\}$……………………………………………………..………………..…1p

3. a) $DE \parallel AB$ și $DF \parallel BE \Rightarrow BEDF$ $paralelogram$……………………………………….1p

$\sphericalangle EBD \equiv \sphericalangle DBF$ si $\sphericalangle DBF \equiv \sphericalangle BDE (alt.int) \Rightarrow \sphericalangle EBD \equiv \sphericalangle BDE \Rightarrow$ ΔEBD $este$ $isoscel$…………………………………………………………………….……….1p

$BEDF - romb \Rightarrow EF \perp BD$…………………………………………………….....………..1p

În ΔMBD, BA$\perp$MD, MF$\perp$BD , BA$\cap$MF=$\{F\}\Rightarrow$ F ortocentrul $\Rightarrow$ DF$\perp$MB...........................1p

$b)$ ΔMDE$\equiv\Delta$MBE (LUL) $\Rightarrow$ m($\sphericalangle$MBC) =90^0. m($\sphericalangle$BCM)=$30^0 \Rightarrow MB = \frac{MC}{2}$………...............1p

ΔBMD $echilateral$, $BA \perp MD \Rightarrow AD = \frac{MB}{2} = \frac{MC}{4}$……………………………………….1p

$S_{\Delta BMC} = \frac{MC\cdot AB}{2} = \frac{4\cdot AD\cdot AB}{2} = 4\cdot S_{\Delta ABD}$...1p

Notă:

Orice alta solutie corecta va fi punctată corespunzator